# Calibrating Fuzzy-Logic-based Rehabilitation Decision Models Using the LTPP Database

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## ABSTRACT
This paper establishes a systematic method to calibrate a fuzzy-logic-based rehabilitation decision model using real cases extracted from the Long Term Pavement Performance (LTPP) database. Fuzzy logic systems are efficient for combining expert knowledge and numeric data. This feature makes it ideal for building decision-support models. A total of eight tables in the LTPP database were used from which to extract pavement and rehabilitation information. The investigation started with 62 rehabilitation cases but only six overlay rehabilitations with thicknesses between 1.5 and 2.5 inches had all the required information and were thus selected to calibrate the decision model. To make the dataset unbiased, six do-nothing cases were created based on the rehabilitation cases. The steepest descent method and back-propagation learning were used to tune the model with the selected rehabilitation events. By reinterpreting the model in the form of neural fuzzy system, the calibration algorithm successfully tuned the decision model to distinguish between rehabilitation cases and do-nothing cases.

## INTRODUCTION
Engineering economic analysis is a widely accepted technique to evaluate and measure the effectiveness of decision options in transportation asset management (*1, 2*). Valuing the costs and impacts of transportation projects in a monetary form is an objective way to assess project-level alternatives, network-level plans, and strategic-level policies. Most economic analysis methods used in transportation asset management, such as benefit cost analysis (BCA) and life cycle cost analysis (LCCA), require the prediction of future project performance and future expenditures to control or correct the deterioration of transportation infrastructures. Maintenance, rehabilitation and reconstruction (MR&R) decisions often consider many factors such as infrastructure design and rehabilitation models, policies, availability of funding, and agency experience (*1*). From an engineer's point of view, the most relevant factors are the infrastructure condition and treatment effectiveness. Studies to predict the condition of pavement, bridges, and other transportation assets are abundant in the literature (*3-5*). However, efforts that summarize and consolidate real life decision-making cases and experiences are rare in the transportation field. In practice, project-level decisions are often based on engineering judgment and experience. With first-hand management experience and the best knowledge of their assets, engineers and asset managers are making decisions everyday regarding pavements, bridges and other assets. Therefore, it would be appropriate to use an agency's specific experience to predict future MR&R activities in engineering

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economic analysis. Properly utilizing the opinion and experience of the asset management staff should lead to more realistic results in economic analysis.

## OBJECTIVE

This paper establishes a systematic method for calibrating the fuzzy-logic-based rehabilitation decision model proposed by Chen et al. (*6*) using real cases extracted from the Long Term Pavement Performance (LTPP) database. The steepest descent method and back-propagation learning algorithm were used to tune the model. To achieve the objective, the following tasks had to be accomplished:

1. Extract reliable cases of overlay treatment from the LTPP database;
2. Identify proper input areas for engineering knowledge and numeric data;
3. Simultaneously tune two fuzzy logic systems with shared membership functions for input variables.

## FUZZY-LOGIC-BASED DECISION MODEL

The decision model to be calibrated in this research was first proposed by Chen et al. (*6*) as a module to schedule rehabilitation activities for LCCA. As shown in Figure 1, the decision model consists of two fuzzy logic systems, namely, Fuzzy Trigger Model and Fuzzy Policy Model. Rules for both models can be customized according to the agency's policies or expert opinions. The trigger model computes the priority of a do-nothing policy, and the policy model evaluates the priority of specific rehabilitation treatments. The LCCA algorithm compares the priorities computed by the two fuzzy logic models each year, and the treatment with the highest priority is selected for that year.

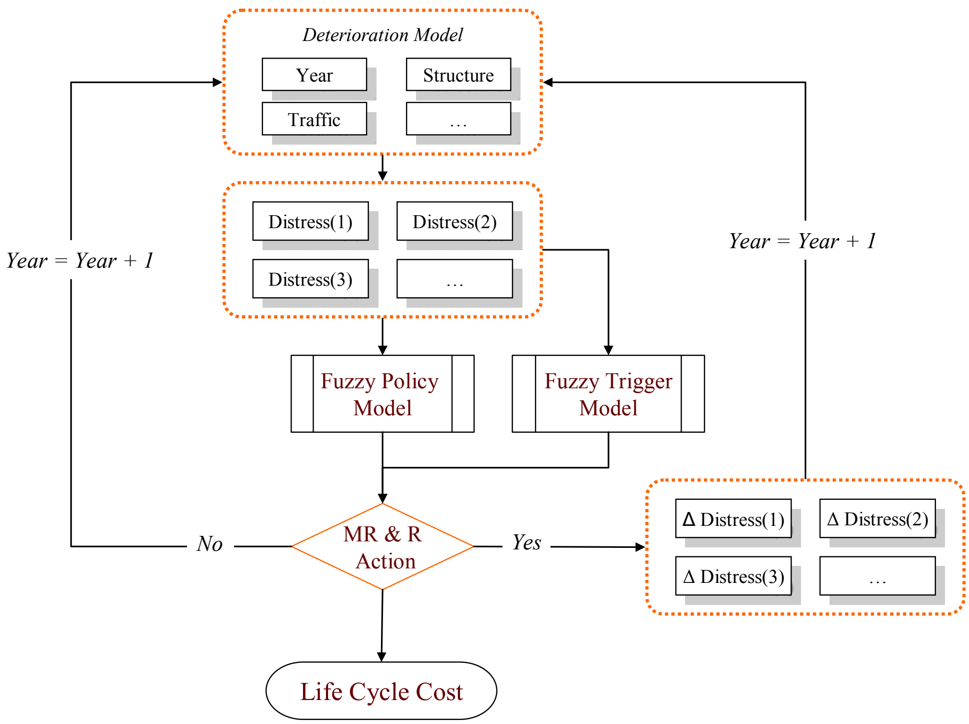


**FIGURE 1 Fuzzy-Logic-based Decision Model (*6*)**

The algorithm's validity relies heavily on the deterioration model and the fuzzy-logic-based decision model. With the help of modern mathematic techniques, including statistical methods and adaptive modeling, the deterioration model could be established and calibrated to be used in a specific geographic region with confidence. However, the project-level decision models have not been comprehensively studied. When reliable history records are not available, the major, if not the only, source of information for constructing the decision models is expert opinions and engineering experience. As objective data becomes available, the decision models can be improved by calibrating these models with the accumulation of real cases, which accurately reflect an agency's MR&R practice. The procedure was used in this research to initially design and then periodically tune the fuzzy-logic-based decision model.

## DATA PREPARATION

The data used to tune the decision model were extracted from the LTPP database. The LTPP database is probably the largest pavement condition data source in the world available for public access. This fact makes it an ideal data source for this research. Figure 2 summarizes the procedure used for extracting the required data from the database. The main steps of this process are elaborated following.

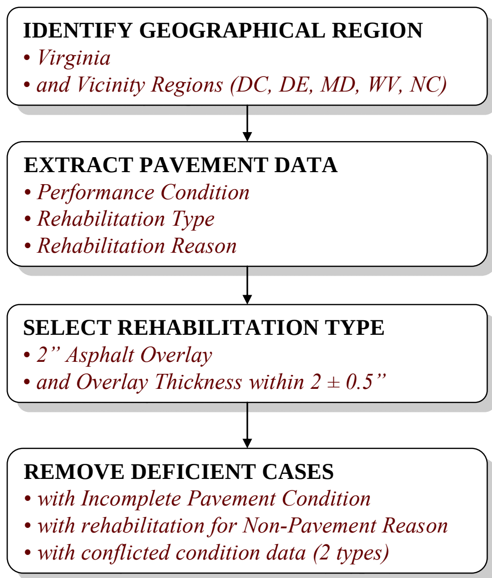


**FIGURE 2 Procedure for Extracting Training Data from LTPP Database**

### Geographical Area of Interest

Since climate and environment have significant impacts on pavement performance and rehabilitation effectiveness, it is inevitable that rehabilitation decisions are also affected by the factors. Therefore, identifying a specific region from which to extract cases would be helpful to reduce the complexity of the model. The state of Virginia is the area of interest in this research. However, to have enough data for the calibration study, five neighboring Mid-Atlantic States (the District of Columbia, Delaware, Maryland, West Virginia, and North Carolina) were also included in the queries.

### Pavement Condition Indicators

Pavement condition is usually measured by the severity and frequency of the distresses preset in the surface. Three types of distresses were considered in the research: alligator cracking, rutting, and roughness. The first two have been widely accepted as important indicators of soundness of pavement structures. Wire-line rut indices were used in this research for rutting measurement because the method usually obtains rutting values with a wider range compared with traditional straight-edge methods. Pavement roughness is important because it is directly noticeable to the traveling public. The decision model was calibrated based on the pavement conditions immediately before a certain type of rehabilitation.

### Rehabilitation Treatments

After reviewing the rehabilitation records of sections in the LTPP database, overlays between 1.5 and 2.5 inches were categorized as one rehabilitation type. This rehabilitation type was referred as thin overlay for this research. Usually, an overlay with thickness in this range is selected to repair pavements that have not suffered severe losses in structural capacity, at which point major rehabilitation or reconstruction has yet become necessary.

### Deficient Records

Not all rehabilitation records were ready to be used to calibrate a decision model capable of reflecting the intended practices. Three types of deficient records were removed from the dataset to be used in calibration. The first group is those with some desired information missing, such as no roughness data collected before rehabilitation. Then, rehabilitations incurred by reasons other than poor pavement conditions are deleted because this research only focuses on decisions due to engineering considerations. The records removed in a last step are those conflicting with their sections' major trend. Typical abnormalities are unexpected performance improvement without any documented rehabilitations.

### Building Dataset for Calibration

Once the rehabilitation events have been identified, section information, rehabilitation type and data, and pavement conditions were extracted from the database to build the dataset. After the cleansing, a total of six cases, as listed in Table 1, were finally used to calibrate the fuzzy-logic-based decision model in this study. What the dataset also needs are cases such that current pavement conditions lead to do-nothing, but any further development of distress would lead to rehabilitation. In this research, the rehabilitation cases were used to create do-nothing cases by reducing the current distresses by 15%. The percentage was selected based on engineering judgment. Adding these do-nothing cases made the dataset balanced.

**TABLE 1 Rehabilitation Cases**

| Case # | State | ID | Alligator Cracking (*%*) | Rutting (*inches*) | Roughness (*inches/mile*) | Overlay Thickness (*inches*) |
|---|---|---|---|---|---|---|
| *1* | MD | 0505 | 18.42% | 0.24 | 116.11 | 2.0 |
| *2* | MD | 0561 | 23.90% | 0.39 | 70.96 | 2.5 |
| *3* | VA | 1002 | 6.15% | 0.20 | 172.74 | 1.5 |
| *4* | VA | 1023 | 4.59% | 0.63 | 121.14 | 1.5 |
| *5* | VA | 1417 | 3.71% | 0.24 | 175.63 | 1.5 |
| *6* | VA | 1423 | 5.09% | 0.28 | 137.36 | 1.5 |

## DESIGN OF FUZZY LOGIC SYSTEMS

Fuzzy logic systems are an extension of the traditional rule-based reasoning (expert systems) that incorporate imprecise, qualitative data in the decision-making process by combining descriptive linguistic rules through fuzzy logic (*7*). A typical fuzzy logic system consists of four components: fuzzifier (input processor), rule base, inference engine, and defuzzifier (output processor). The fuzzifier and defuzzifier provide mapping between crisp values and fuzzy sets. The concept of fuzzy sets provides the basis of fuzzy logic systems. There have been many references that explain the concept (*8, 9*). A fuzzy set is a set whose members have an associated membership function value between 0 and 1, while the traditional crisp sets only allow its members to take value of 0 or 1. The rule base is where the knowledge of models is stored. The knowledge needed to build the rules can be provided by experts, extracted from collected data, or both. A form of IF-THEN clauses is often used to store the knowledge. The inference engine determines how the rules are triggered and processed to map input values into outputs. For simplicity, the fuzzy logic systems studied in this research were designed using singleton fuzzification, Gaussian membership functions, product implication and t-norm operations, and height defuzzification. The following discussion will focus on this specific architecture unless indicated otherwise.

### Membership Functions

Gaussian functions are used in this paper to define the membership of each element in a fuzzy set. So for a single input value of $x$, its fuzzy membership is given in equation (1).

$$\mu_{F_k^l}(x) = \exp\left(-\frac{(x - m_{F_k^l})^2}{2\sigma_{F_k^l}^2}\right) \quad i = 1, \ldots, N \qquad (1)$$

Where $m_{F_k^l}$ = the mean value of Gaussian function of the *k*th input in the *l*th rule;

$\sigma_{F_k^l}$ = the standard deviation of Gaussian function of the *k*th input in the *l*th rule.

The number of membership function terms for a specific input or output variable depends on the previous experience about how many categories are enough. The simplest configuration (used in this research) is categorizing each pavement distress condition into two groups: good or poor. For example, the rutting condition was defined by two membership functions, one for good rutting condition and one for poor. A specific rutting value could partially belong to good and poor at the same time in fuzzy logic systems.

**Inference Rules**

The Rule-bases store the knowledge of fuzzy logic systems. To fully utilize available expert knowledge, the proposed tuning method starts by defining inference rules. If a fuzzy logic system has $p$ inputs $x_1$, $x_2$, …, $x_p$, one output $y$, and $M$ inference rules, then the $l$th rule has the form:

$$R^l\text{: IF } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_p \text{ is } F_p^l\text{, THEN is } y \text{ is } G^l;\ l = 1, \ldots, M$$

Where $F_k^l$ is the membership function term of the *k*th input ($k = 1, \ldots, p$) in the *l*th rule and $G^l$ is the membership function term of the output.

Since the decision model has three distress conditions as inputs, at least eight inference rules are required to fully cover all possible premise conditions. Table 2 gives the inference rules and initial mean values of output membership functions of the two fuzzy logic systems utilized in this research. If all three conditions are good, the do-nothing option is of the highest priority, 1, and the rehabilitation option is of the lowest priority, 0. As the pavement conditions deteriorate, the priorities of do-nothing decrease while those of rehabilitation increase. Finally, when all three conditions are poor, rehabilitation gains an overwhelming priority, 1, over do-nothing, 0.

**TABLE 2 Inference Rule Bases of the Decision Model**

| Rule | Alligator Cracking | Rutting | Roughness | Do-Nothing Priority | Rehabilitation Priority |
|---|---|---|---|---|---|
| | *Premise* | | | *Consequences* | |
| *1* | Good | Good | Good | 1 | 0 |
| *2* | Poor | Good | Good | 0.75 | 0.25 |
| *3* | Good | Poor | Good | 0.75 | 0.25 |
| *4* | Good | Good | Poor | 0.75 | 0.25 |
| *5* | Poor | Poor | Good | 0.25 | 0.75 |
| *6* | Poor | Good | Poor | 0.25 | 0.75 |
| *7* | Good | Poor | Poor | 0.25 | 0.75 |
| *8* | Poor | Poor | Poor | 0 | 1 |

The logic behind this rule base is valid from the engineering perspective; do nothing to pavements in good condition and rehabilitate pavements in poor condition. However, the challenge is to find the correct value for priorities in each rule and the parameters, mean values and standard deviations, of the various membership functions to make the decision model consistent with previous practices. To achieve this ultimate goal, it is necessary to have a systematic procedure to calibrate the model using real cases. The concept of fuzzy basis function provides a bridge between fuzzy logic system and mathematic programming.

**Fuzzy Basis Functions**

The concept of fuzzy basis functions was first introduced by Wang and Mendel (*10*). Their objective was to develop a mathematical formula representing how a crisp input $\mathbf{x}$ is mapped into a crisp output $y = f(\mathbf{x})$ through fuzzy logic systems. Let $\overline{y}^l$ denote the mean value of output membership function in the $l$th rule. The mapping formula was constructed as follows:

$$y = f(\mathbf{x}) = \sum_{l=1}^{M} \overline{y}^l \phi_l(\mathbf{x}) \quad l = 1, \ldots, M \tag{2}$$

Where $\phi_l(\mathbf{x})$ is the fuzzy basis function which is given based on specific choices for fuzzifier, membership functions, composition, implication, t-norm and defuzzifier. For the architecture used in this study, the fuzzy basis function is:

$$\phi_l(\mathbf{x}) = \frac{\prod_{k=1}^{p} \mu_{F_k^l}(x_k)}{\sum_{l=1}^{M} \prod_{k=1}^{p} \mu_{F_k^l}(x_k)} \tag{3}$$

Where $\mu_{F_k^l}(x_k)$ is the value of membership of the $k$th input ($k = 1, \ldots, p$) in the $l$th rule ($l = 1, \ldots, M$).

**Tuning Fuzzy Logic Systems**

Based on the idea of steepest descent method, Mendel (*9*) developed a method to tune the antecedent and consequent parameters in fuzzy logic systems. This method was used as the basis of the calibration algorithm used in the paper and is briefly explained in this section.

If $N$ groups of paired input-output data are available to tune a fuzzy logic system and $\mathbf{x}$ denotes the inputs and $y$ for output, the cost function for the $i$th group can be defined as follows:

$$e^{(i)} = \frac{\left[f_s(\mathbf{x}^{(i)}) - y^{(i)}\right]^2}{2} \quad i = 1, \ldots, N \tag{4}$$

Following the principle of steepest descent methods, three equations were developed to minimize $e^{(i)}$ ($i = 1, \ldots, N$, $l = 1, \ldots, M$ and $k = 1, \ldots, p$):

$$m_{F_k^l}(i+1) = m_{F_k^l}(i) - \alpha_m \left[f_s(\mathbf{x}^{(i)}) - y^{(i)}\right]\left[\overline{y}^l(i) - f_s(\mathbf{x}^{(i)})\right] \times \frac{\left[x_k^{(i)} - m_{F_k^l}(i)\right]}{\sigma_{F_k^l}^2(i)} \phi_l(\mathbf{x}^{(i)}) \tag{5}$$

$$\overline{y}^l(i+1) = \overline{y}^l(i) - \alpha_{\overline{y}} \left[f_s(\mathbf{x}^{(i)}) - y^{(i)}\right] \phi_l(\mathbf{x}^{(i)}) \tag{6}$$

$$\sigma_{F_k^l}(i+1) = \sigma_{F_k^l}(i) - \alpha_\sigma \left[f_s(\mathbf{x}^{(i)}) - y^{(i)}\right]\left[\overline{y}^l(i) - f_s(\mathbf{x}^{(i)})\right] \times \frac{\left[x_k^{(i)} - m_{F_k^l}(i)\right]^2}{\sigma_{F_k^l}^3(i)} \phi_l(\mathbf{x}^{(i)}) \tag{7}$$

Because $\overline{y}^l$, $m_{F_k^l}$ and $\sigma_{F_k^l}$ are parameters associated with membership functions for physically meaningful quantities, it is usually possible to obtain very good initial values for them. Choosing them smartly will help this algorithm to converge much faster (*11*). The learning parameters, $\alpha_m$, $\alpha_{\overline{y}}$ and $\alpha_\sigma$ must also be chosen with some care.

**MODEL CALIBRATION**

A challenge associated with the tuning of the decision model in Figure 1 is how to simultaneously optimize the design of two fuzzy logic systems. Since they share the same input variables, it will be ideal if they would share the definition of membership functions of each input variable. Using the same language in the two fuzzy logic systems facilitates expert knowledge acquisition and makes the model easier to understand and accept by decision makers. However, this also means one more objective and several more constraints in the calibration phase, which usually makes the nonlinear programming problem more difficult to solve.

Fortunately, shared membership functions also provide direct connections between the two systems. With these connections, the competition among two fuzzy logic systems can be reinterpreted as an integrated neural fuzzy system. As shown in Figure 3, a feed-forward multilayer network was constructed to describe the decision model in terms of a neural fuzzy system. The membership functions and inference rules connected by solid lines are components of the trigger model and those connected by broken lines belong to the policy model.

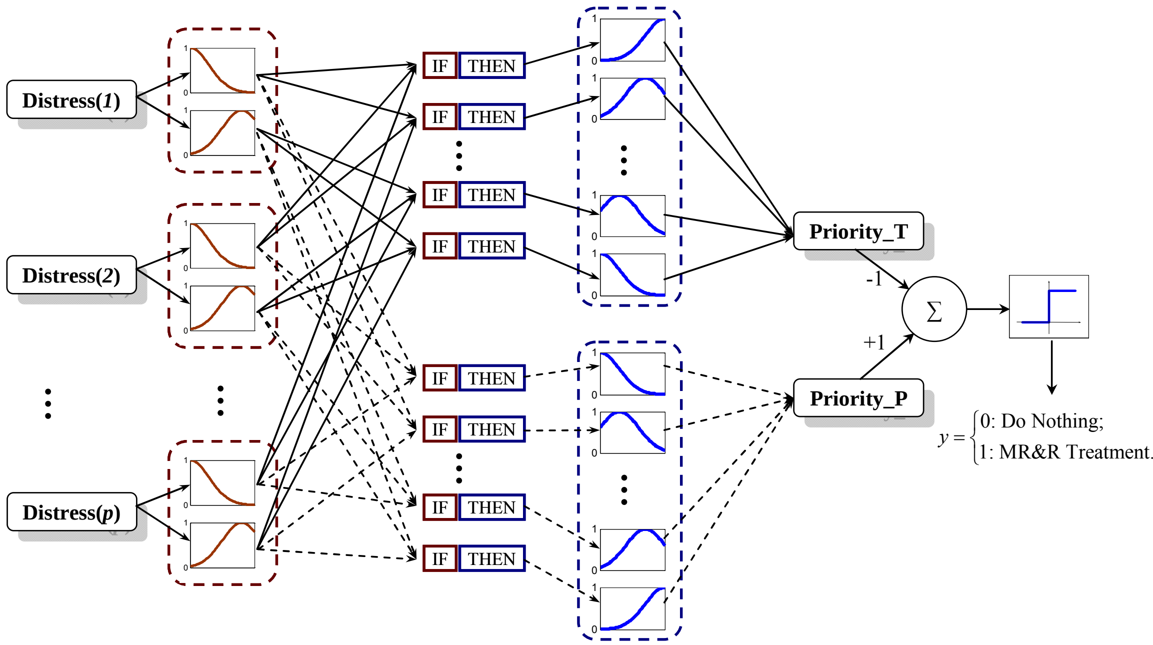


**FIGURE 3 Reinterpreting the Decision Model in the form of a Neural Fuzzy System**

Under this network structure, the input nodes represent transportation infrastructure conditions; the hidden layers consist of nodes functioning as membership functions, fuzzy logic rules, and comparison of priorities; and the output layer represents final decisions. The detailed explanation of the layers is as follows:

- *Input*: Input nodes represent current condition of transportation assets. They are measured by the extent or magnitude of the distresses considered.
- *Hidden Layer 1*: Each single node in this layer performs one membership function for a certain input variable, i.e., a distress type in this study. Therefore, links between layer 1 and input nodes only occur when the nodes are designated to a specific distress type.
- *Hidden Layer 2*: This layer represents the inference rules designed for the two fuzzy logic models. It is possible to have multiple inputs for nodes of this layer when the represented rule has multiple antecedents.
- *Hidden Layer 3*: Nodes in this layer are essentially the membership functions for the output variables, the priorities of do-nothing or rehabilitation. Together with layer 2, this layer functions as the inference engine of the two fuzzy logic models.
- *Hidden Layer 4*: The layer combines the outputs from the fuzzy logic rules in each of the two models. A defuzzification method has to be pre-specified in the initial design phase.

- *Output*: The output layer gives the final recommended treatment. It compares the two output priorities and selects a winner, i.e., the treatment with the higher priority. Therefore, the weight of link is 1 between output node and the priority of the policy model and -1 for the priority of the trigger model.

The five-layer neural fuzzy system makes it possible to use neural network learning techniques for this research.

**Calibrating the Decision-support Models**

With the network-structured decision model in Figure 3, it is a natural choice to use a back-propagation (BP) learning algorithm to do the calibration work. The concept of BP learning was one of the most important developments in neural networks (*12, 13*). In the algorithm, the error(s) at the output layer is propagated backward to adjust the connection weights of preceding layers to minimize output errors. After a number of iterations, multilayer feed-forward networks can be systematically optimized by this algorithm.

Mathematically, the outputs are 0 for do-nothing or 1 for thin-overlay in this research. When pavement conditions are deteriorated to a level just severe enough for triggering thin overlay rehabilitation, the priority from the policy model is desired to be slightly lower than the priority from trigger model. In other words, for the pavement conditions used in calibration, the outputs from two fuzzy logic systems expected to be very close but still capable of differentiating do-nothing cases from rehabilitation cases. Based on this consideration, the error function was formulated as:

$$e^{(i)} = (d^{(i)} - f^{(i)})\left|\text{Priority}^{(i)}_{policy} - \text{Priority}^{(i)}_{trigger}\right| \tag{8}$$

Where $d^{(i)}$ = the desired decision based on the *i*th pair of pavement conditions;
$f^{(i)}$ = the decision by decision model based on the *i*th pair of pavement conditions;
$\text{Priority}^{(i)}_{policy}$ = the priority of rehabilitation produced by policy model;
$\text{Priority}^{(i)}_{trigger}$ = the priority of do-nothing produced by trigger model;

The errors calculated by equation (8) are for the entire neural fuzzy system. Following the principle of back-propagation method, the error propagated to trigger model is $-e^{(i)}$; the error propagated to policy model is $e^{(i)}$. The calibration then becomes tuning the two fuzzy logic systems. As shown in Figure 4, a total of 72 epochs were sufficient to get the decision model to converge. Table 3 gives the values of the parameters of the decision model before and after calibration.

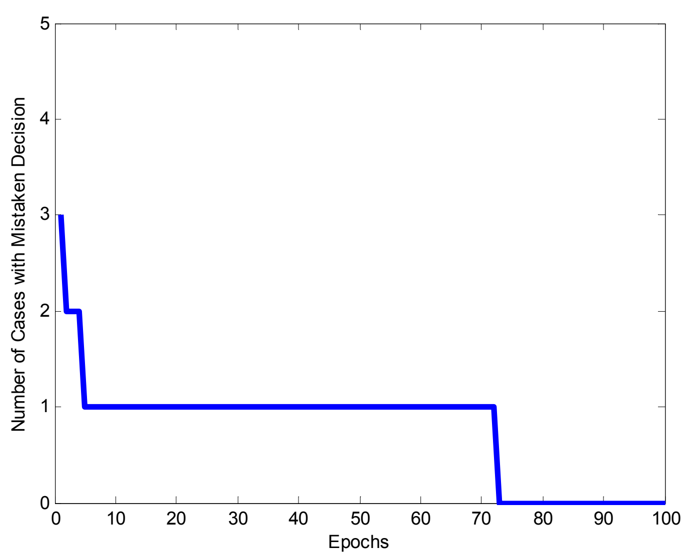


**FIGURE 4 Number of cases with mistaken decisions decreasing through multi-pass calibration**

**TABLE 3 Fuzzy-Logic-based Decision Model**

| Variable | Membership Function | Initial Values | | Values after Tuning | |
|---|---|---|---|---|---|
| | | *M* | *Σ* | *m* | *σ* |
| Alligator Cracking | *Good* | 5% | 10% | 0% | 11% |
| | *Poor* | 25% | 10% | 17% | 18% |
| Rutting | *Good* | 0.1 | 0.3 | 0.07 | 0.32 |
| | *Poor* | 1 | 0.3 | 0.92 | 0.45 |
| Roughness | *Good* | 50 | 50 | 50 | 50 |
| | *Poor* | 200 | 50 | 200 | 50 |
| Do-Nothing Priority | *Rule 1* | 1 | - | 0.88 | - |
| | *Rule 2* | 0.75 | - | 0.60 | - |
| | *Rule 3* | 0.75 | - | 0.77 | - |
| | *Rule 4* | 0.75 | - | 0.75 | - |
| | *Rule 5* | 0.25 | - | 0.30 | - |
| | *Rule 6* | 0.25 | - | 0.25 | - |
| | *Rule 7* | 0.25 | - | 0.25 | - |
| | *Rule 8* | 0 | - | 0.00 | - |
| Rehabilitation Priority | *Rule 1* | 0 | - | 0.12 | - |
| | *Rule 2* | 0.25 | - | 0.40 | - |
| | *Rule 3* | 0.25 | - | 0.23 | - |
| | *Rule 4* | 0.25 | - | 0.25 | - |
| | *Rule 5* | 0.75 | - | 0.70 | - |
| | *Rule 6* | 0.75 | - | 0.75 | - |
| | *Rule 7* | 0.75 | - | 0.75 | - |
| | *Rule 8* | 1 | - | 1.00 | - |

## SUMMARY AND CONCLUSIONS

This paper proposes a systematic method to calibrate a fuzzy-logic-based rehabilitation decision model using real cases extracted from the Long Term Pavement Performance (LTPP) database. The initial architecture of fuzzy-logic-based decision model was designed based on engineering knowledge; inference rules were developed to cover all possible combinations of the three pavement distresses considered. The decision-support model was tuned to distinguish between rehabilitation cases and do-nothing cases by reinterpreting the model as a neuron-fuzzy system. The application of the proposed method in a simple example demonstrated that the approach is feasible and practical. The results of the investigation suggest that accumulated numeric data can help study the decision behavior of a transportation agency. If properly designed and maintained, a DOT asset management database stores the corporate knowledge of the agency. Rule-based fuzzy logic systems provided a user-friendly platform to elicit experts' opinions and agency practices. Rules with the form of "IF-THEN" are easily understood by engineering and managing professionals. By reinterpreting the model in the form of a neuro-fuzzy system, the calibration algorithm takes advantage of the learning capabilities of artificial neural networks for tuning the fuzzy membership functions and rules. With reasonable efforts, the initial model was tuned to reflect the rehabilitation decisions made on the LTPP sections.

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